A Flexible Model with Linear Approximation for Left Truncated Right Censored Data

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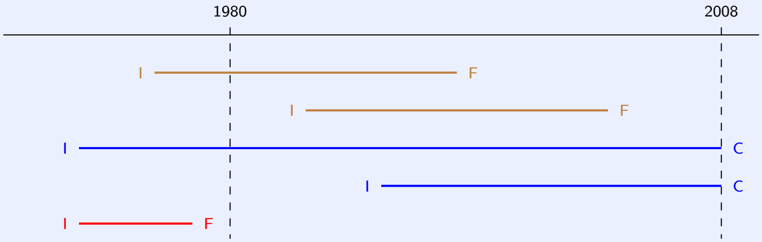
Main Sections

1. Left Truncation and Right Censoring
2. Our Model
3. Likelihood Inference
4. Robustness Study
5. Data Analysis
6. Conclusion

Left Truncation and Right Censored Data

- Left truncated and right censored data (LTRC).



LTRC Data: Another example

- Lifetimes were collected from a retirement centre, Channing House, in Palo Alto, California.
- Minimum age of to enter the centre: 60 years
- Individuals entered the centre at different ages after 60 years.
- Starting time of study: 1964.
- Ending time of study: July 01, 1975.
- Total individuals: 462 (F-365, M-97).
- No information before entering centre $\rightarrow$ Left truncation.
- Left truncation time: Age of entering.
- Alive individuals left house before ending time or still alive at the end of the study $\rightarrow$ Right censoring.
- Censoring time: Age at departure or at July 01, 1975.
- Censored individuals: 286 (F-235, M-51).

LTRC Data: Another example

- Available in R-package `boot`.
- First 6 rows of the data is given below.

Sex	Entry	Exit	Time	Cens
Male	782	909	127	1
Male	1020	1128	108	1
Male	856	969	113	1
Male	915	957	42	1
Male	863	983	120	1
Male	906	1012	106	1

Brief Literature Review

- Hong et al. (2009)[1] : Weibull Distribution.

[1] Hong Y., Meeker W. Q., and McCalley J. D. (2009). Prediction of remaining life of power transformers based on left truncated and right censored lifetime data, *The Annals of Applied Statistics* 3:857–879.

Brief Literature Review

- Balakrishnan and Mitra (2012)[2]: Weibull Distribution, used EM algorithm.

[2] Balakrishnan N. and Mitra D. (2012). Left truncated and right censored Weibull data and likelihood inference with an illustration, *Computational Statistics and Data Analysis*, 56:4011–4025

Brief Literature Review

- Balakrishnan and Mitra (2014)[3] : Generalized gamma family of distributions, EM algorithm, model discrimination.
- Mitra et al. (2021)[4] : Lehmann family of distribution, stochastic EM, model discrimination.

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[3] Balakrishnan, N. and Mitra, D. (2014). EM-based likelihood inference for some lifetime distributions based on left truncated and right censored data and associated model discrimination (with discussions). *South African Statistical Journal*, 48, 125–204.

[4] Mitra, D., Kundu, D. and Balakrishnan, N. (2021). Likelihood analysis and stochastic EM algorithm for left truncated right censored data and associated model selection from the Lehmann family of life distributions. *Japanese Journal of Statistics and Data Science*, 4, 1019–1048.

Brief Literature Review

- Emura and Michimae (2022)[5] : Review of parametric models and inferences.

[5] Emura, T., and Michimae, H. (2022). Left-truncated and right-censored field failure data: Review of parametric analysis for reliability. Quality and Reliability Engineering International, 38, 3919–3934.

Covariates

- Presence of co-variates.
- Channing House data: Male or Female.
- Cox's proportional hazards model:

$$\lambda(t, \boldsymbol{x}) = \lambda_0(t) \exp\left(\boldsymbol{x}'\beta\right).$$

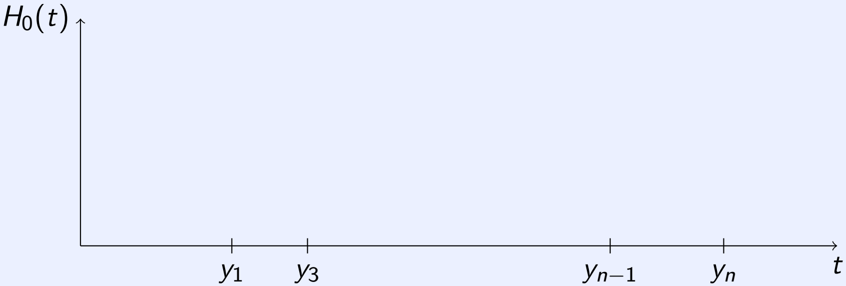
- In terms of cumulative hazard function:

$$\Lambda(t, \boldsymbol{x}) = \Lambda_0(t) \exp\left(\boldsymbol{x}'\beta\right).$$

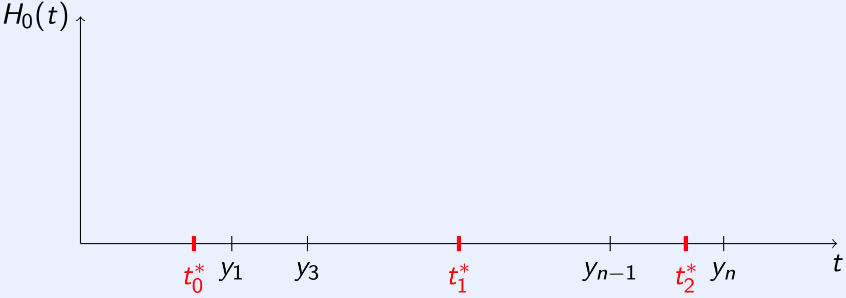
- Parametric or non-parametric estimate.
- Houwelingen and Stijnen (2014)[6] .

[6] van Houwelingen, H. C. and Stijnen, T. (2014). Cox regression model. In: Handbook of Survival Analysis, Eds: Klein, J. P., van Houwelingen, H. C., Ibrahim, J. G., and Scheike, T. H. CRC Press, Boca Raton, FL.

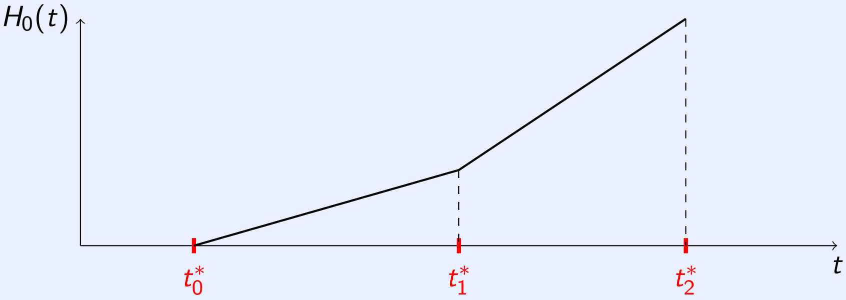
Piecewise Linear Approximation



Piecewise Linear Approximation



Piecewise Linear Approximation



Piecewise Linear Approximation

- Lifetimes: $y_1, y_2, \ldots, y_n$ (observed failures and right censored).
- Truncation times: $\tau_1, \tau_2, \ldots, \tau_n$.
- Cut-points: $t_0^*, t_1^*, \ldots, t_n^*$.
- $t_0^* < t_1^* < \ldots < t_n^*$, $t_0^* \leq \min\{y_{min}, \tau_{min}\}$ and $t_n^* \leq y_{max}$.
- Approximate $\Lambda_0(\cdot)$ by $H_0(\cdot)$:

$$H_0(t) = \sum_{k=1}^{N} (a_k + b_k t) I\left(t_{k-1}^* \leq t < t_k^*\right).$$

- $H_0(t_0^*) = 0 \implies a_1 = 0$.
- $H_0(\cdot)$ increasing $\implies b_k > 0$ for all k.
- $H_0(\cdot)$ continuous at t_k^* for all $k \implies a_k$ can be expressed in terms of b_k's.

Likelihood Inference

- Approximate cumulative hazard function:

$$H(t, \boldsymbol{x}) = H_0(t) \exp\left(\boldsymbol{x}'\beta\right).$$

- Obtain log-likelihood function.
- Number of parameters to estimate:
  Number of cut-points - 1 + Number of co-variates.
- Maximize numerically to obtain MLEs.
- Asymptotic confidence intervals of parameters.

Choice of Cut-points

- Looking at Breslow estimate of cumulative baseline hazard.
- Some suitable quantiles of data.
- AIC / BIC.

Robustness Study

- Simulation based study.
- To check if proposed model fits well to data generated from different parametric models.
- Generate data from a model.
- Estimate based on proposed model.
- Compare closeness.

Robustness Study

- Absolute integrated error (AIE):

$$AIE(x) = \frac{1}{R} \sum_{k=1}^{R} \frac{1}{y_{max,k} - y_{min,k}} \int_{y_{min,k}}^{y_{max,k}} \left| \widehat{H}_{PLA,k}(t,x) - \Lambda(t,x) \right| dt.$$

- Two parent models are considered:
  - Weibull distribution:

$$\Lambda(t, x) = (\lambda t)^{\gamma} e^{-\beta x}.$$

  - PH model with a mixture of Weibull distributions as the cumulative baseline hazard:

$$\Lambda(t, x) = (\lambda_1 t^{\alpha_1} + \lambda_2 t^{\alpha_2}) e^{-\beta x}.$$

Robustness Study

- Single covariate with two levels: 0 and 1.
- Different censoring percentage: 20%, 40%
- Different truncation percentage: 10%, 30%
- Different values of parameters.
- AIE values are not very large (< 0.5).
- PLA-based model is less affected by truncation percentage.
- PLA-based model is more affected by censoring percentage.

Analysis of Channing House Data

- The first 6 rows of data:

Sex	Entry	Exit	Time	Cens
Male	782	909	127	1
Male	1020	1128	108	1
Male	856	969	113	1
Male	915	957	42	1
Male	863	983	120	1
Male	906	1012	106	1

- Sex is used as covariate (Female: $x = 1$, Male: $x = 0$).
- Time transformation: $\frac{t-720}{100}$.

Analysis of Channing House Data

Cut Points	b_1	b_2	b_3	b_4	β	AIC
t_0^*, $q_{0.2}$, $q_{0.5}$, $q_{0.7}$	0.298	0.387	1.161	1.353	-0.306	542.859
t_0^*, $q_{0.2}$, $q_{0.5}$, $q_{0.8}$	0.298	0.387	1.204	1.391	-0.307	542.917
t_0^*, $q_{0.3}$, $q_{0.5}$, $q_{0.7}$	0.333	0.353	1.163	1.354	-0.307	544.014
t_0^*, $q_{0.3}$, $q_{0.5}$, $q_{0.8}$	0.333	0.354	1.205	1.392	-0.309	544.071
t_0^*, $q_{0.2}$, $q_{0.5}$	0.298	0.387	1.273	–	-0.307	541.443
t_0^*, $q_{0.3}$, $q_{0.5}$	0.333	0.353	1.275	–	-0.308	542.597
t_0^*, $q_{0.5}$, $q_{0.7}$	0.340	1.163	1.354	–	-0.308	542.065
t_0^*, $q_{0.5}$, $q_{0.8}$	0.340	1.206	1.393	–	-0.309	542.122

Analysis of Channing House Data

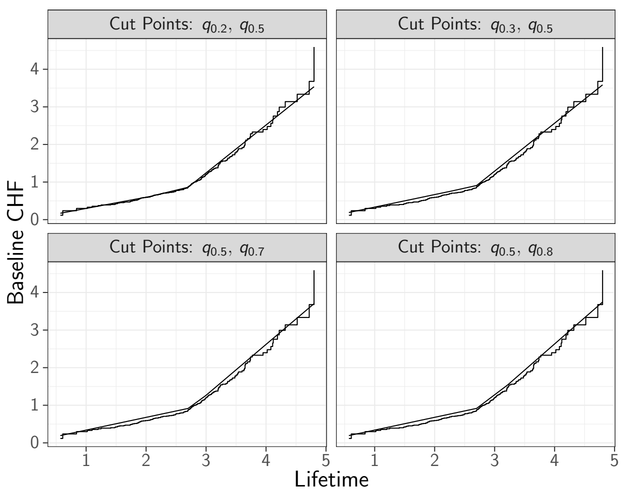


Figure: Plot of estimated baseline CHF with two cut points

Analysis of Channing House Data

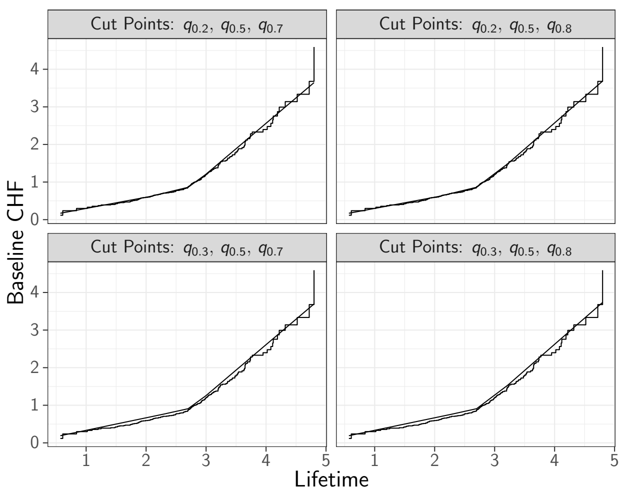


Figure: Plot of estimated baseline CHF with three cut points

Concluding Remark

- PLA based model for predictions of future failures.
  - Breslow estimator is constructed over the observed range of data.
  - PLA based model can be extended beyond the observed data range for making predictions.
- PLA-based model for infant failures.
  - Significant mass at numerical zero.
  - a_1 (the intercept of the first linear piece) > 0.
  - Estimate a_1 based on data.

Thank You