

Indian Institute of Technology Guwahati
Statistical Inference (MA682)
Problem Set 06

1. Let X_1, \dots, X_n be a RS from an exponential distribution with location parameter θ . That is the PDF of X_i 's are given by $f(x) = e^{-(x-\theta)}I_{(\theta, \infty)}(x)$. Find a level α likelihood ratio test for the null hypothesis $H_0 : \theta \leq \theta_0$ against the alternative hypothesis $H_1 : \theta > \theta_0$, where θ_0 is a given real number.

2. Suppose that X_1, X_2, \dots, X_n are *i.i.d.* random variables from a exponential probability density function

$$f(x; \theta) = \theta^{-1}e^{-x/\theta}I_{(0, \infty)}(x),$$

where $\theta > 0$ is assumed unknown. With preassigned $\alpha \in (0, 1)$, derive a level α likelihood ratio test for $H_0 : \theta = \theta_0 (> 0)$ against $H_1 : \theta \neq \theta_0$.

3. Let X_1, X_2, \dots, X_n be a RS from $N(\mu_1, \sigma^2)$ distribution and Y_1, Y_2, \dots, Y_m be a RS from $N(\mu_2, \sigma^2)$ distribution. Also, assume that X_i 's are independent of Y_j 's. Derive a level α LRT for $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 \neq \mu_2$.

4. Let X_1, X_2, \dots, X_n be a RS from $N(\mu_1, \sigma_1^2)$ distribution and Y_1, Y_2, \dots, Y_m be a RS from $N(\mu_2, \sigma_2^2)$ distribution. Also, assume that X_i 's are independent of Y_j 's. Derive a level α LRT for $H_0 : \sigma_1^2 = \sigma_2^2$ against $H_1 : \sigma_1^2 \neq \sigma_2^2$.

5. Let $(X_1, Y_1), (X_2, Y_2), \dots, (X_n, Y_n)$ be a random sample from a bivariate normal distribution $N_2(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. Derive a level α LRT for

(a) $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 > \mu_2$.

(b) $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 < \mu_2$.

(c) $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 \neq \mu_2$.

6. Let $(X_1, Y_1), (X_2, Y_2), \dots, (X_n, Y_n)$ be a random sample of size $n \geq 3$ from a bivariate normal distribution $N_2(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. Derive a level α LRT for $H_0 : \rho = 0$ against $H_1 : \rho \neq 0$. You may use the fact that

$$\frac{r\sqrt{n-2}}{\sqrt{1-r^2}} \sim t_{n-2} \quad \text{if } \rho = 0.$$