

**Indian Institute of Technology Guwahati**  
**Statistical Inference (MA682)**  
**Problem Set 05**

1. Let  $\phi(\cdot)$  be a most powerful level  $\alpha$  test for testing  $H_0 : \theta = \theta_0$  against  $H_1 : \theta = \theta_1$ . Then show that  $\beta(\theta_0) \leq \beta(\theta_1)$ , where  $\beta(\cdot)$  is the power function of the most powerful test.
2. Let  $X_1, X_2, \dots, X_n$  be a random sample from a  $N(\mu, \sigma^2)$  distribution, where  $\sigma$  is known.
  - (a) Find MP level  $\alpha$  test for  $H_0 : \mu = \mu_0$  against  $H_1 : \mu = \mu_1$ , where  $\mu_1 < \mu_0$ .
  - (b) Find UMP level  $\alpha$  test for  $H_0 : \mu = \mu_0$  against  $H_1 : \mu < \mu_0$ .

3. Let  $X_1, X_2, \dots, X_n$  be a random sample from the PDF

$$f(x, \delta, b) = \frac{1}{b\Gamma(\delta)} x^{\delta-1} e^{-\frac{x}{b}} \quad \text{if } x > 0,$$

where both  $b > 0$  and  $\delta > 0$  are unknown. Derive MP level  $\alpha$  test for  $H_0 : b = b_0, \delta = \delta^*$  against  $H_1 : b = b_1, \delta = \delta^*$ , where  $b_1 > b_0$ .

4. Let  $X_1, X_2, \dots, X_n$  be a random sample from a  $P(\lambda)$ , where  $\lambda > 0$ . Find the most powerful level  $\alpha$  test for  $H_0 : \lambda = \lambda_0$  against  $H_1 : \lambda = \lambda_1 (> \lambda_0)$ .
5. Let  $X_1$  and  $X_2$  be a random sample of size two from a probability density function  $f(x), x \in \mathbb{R}$ . Consider the following two functions

$$f_0(x) = \frac{3}{64} x^2 I_{(0,4)}(x) \quad \text{and} \quad f_1(x) = \frac{3}{16} \sqrt{x} I_{(0,4)}(x).$$

Determine the most powerful level  $\alpha$  test for testing  $H_0 : f(x) = f_0(x)$  against  $H_1 : f(x) = f_1(x)$ .

6. Let  $X_1, X_2$  be independent random variables distributed as  $N(\mu, \sigma^2)$  and  $N(\mu, 4\sigma^2)$ , respectively. Suppose that  $\mu \in \mathbb{R}$  is unknown, but  $\sigma > 0$  is known. Derive MP level  $\alpha$  test for  $H_0 : \mu = \mu_0$  against  $H_1 : \mu = \mu_1 (> \mu_0)$ . Note that the random variables are independent but not identically distributed.