**Indian Institute of Technology Guwahati**
**Statistical Inference (MA682)**
**Problem Set 04**

1. A sample of size 25 from a normal population with variance 81 produced a sample mean 81.2. Find a 95% confidence interval for the mean.

2. A sample of size 25 from a normal population produced a sample mean 81.2 and sample variance 81. Find a 95% confidence interval for the mean.

3. Let $\overline{X}$ be the mean of a random sample of size $n$ from $\mathrm{N}(\mu, 16)$. Find the smallest sample size $n$ such that $(\overline{X} - 1, \overline{X} + 1)$ is a 90% confidence interval for $\mu$.

4. Let $X_1, X_2, \ldots, X_m$ and $Y_1, Y_2, \ldots, Y_n$ be independent random samples from $\mathrm{N}(\mu_1, \sigma^2)$ and $\mathrm{N}(\mu_2, \sigma^2)$, respectively. Find a $100(1-\alpha)\%$ confidence interval for $\mu_1 - \mu_2$ when (a) $\sigma$ is known, and (b) $\sigma$ is unknown.

5. Two independent samples, each of size 7, from normal populations with common unknown variance $\sigma^2$ produced sample means 4.8 and 5.4 and sample variances 8.38 and 7.62, respectively. Find a 95% confidence interval for the difference between the means of samples 1 and 2.

6. Let $X_1, X_2, \ldots, X_n$ be identically and independently distributed random variables with common probability density function

$$f(x; \theta) = e^{-(x-\theta)} I_{(\theta, \infty)}(x).$$

   Find a pivot and construct $100(1-\alpha)\%$ lower, upper, and symmetric confidence interval. Also find the smallest length $100(1-\alpha)\%$ confidence interval for $\theta$ based on the pivot.

7. For a sample of size one from the population

$$f(x; \theta) = \frac{2}{\theta^2}(\theta - x) I_{(0, \theta)}(x),$$

   Find the $100(1-\alpha)\%$ symmetric confidence interval for $\theta > 0$ using pivotal technique.