Indian Institute of Technology Guwahati
Statistical Inference (MA682)
Problem Set 01

1. Suppose that the random variable X can take on any of the values 1, 2, ..., 10 with respective probabilities 0.06, 0.06, 0.06, 0.06, 0.06, 0.15, 0.13, 0.14, 0.15, 0.13. Give an algorithm that generates the values from the distribution of X.

2. Present a method to generate the value of X, where

$$P(X = j) = \left(\frac{1}{2}\right)^{j+1} + \frac{1}{4}\left(\frac{2}{3}\right)^{j} \quad \text{for} \quad j = 1, 2, 3, \ldots.$$

   [Hint: Look for generation from mixture distribution.]

3. Give a method for generating random numbers form the PDF

$$f(x) = \frac{e^x}{e-1} \quad \text{for} \quad 0 < x < 1.$$

4. Give a method for generating random numbers form the PDF

$$f(x) = \begin{cases} \frac{x-2}{2} & \text{if } 2 < x < 3 \\ \frac{6-x}{6} & \text{if } 3 \leq x < 6. \end{cases}$$

5. Give a method for generating random numbers form the CDF

$$F(x) = 1 - e^{-\alpha x^{\beta}} \quad \text{for} \quad x > 0,$$

   where $\alpha > 0$ and $\beta > 0$.

6. Give a method for generating random numbers form the PDF

$$f(x) = \begin{cases} e^{2x} & \text{if } x < 0 \\ e^{-2x} & \text{if } x \geq 0. \end{cases}$$

7. Give a method to generate a random number having CDF

$$F(x) = \int_0^{\infty} x^y e^{-y} dy \quad \text{for} \quad 0 < x < 1.$$

   [Hint: Consider the conditional CDF of X given $Y = y$ as $P(X \leq x | Y = y) = x^y$, $0 < x < 1$ and the marginal distribution of Y as exponential with mean 1.]

8. Suppose that it is easy to generate random numbers from the CDFs F_1, F_2, ..., F_n. How can we generate from the following CDFs?

   (a) $F(x) = \prod_{i=1}^{n} F_i(x)$.

(b) $F(x) = 1 - \prod_{i=1}^{n} (1 - F_i(x))$.

9. Let G be a CDF with PDF g and suppose that for constants $a < b$, we want to generate from the CDF

$$F(x) = \frac{G(x) - G(a)}{G(b) - G(a)} \quad \text{for} \quad a \leq x \leq b.$$

   (a) If X has CDF G, then F is the conditional CDF of X given what information?

   (b) Show that the acceptance-rejection method reduces in this case to generate a random variable X having distribution G and the accepting it if it lies between a and b.

10. Let X be an exponential random variable with mean 1. Give an efficeint algorithm for simulating random number from the conditional distribution of X given $X < 0.05$. [Hint: Find the CDF corresponding to the conditional distribution]

11. Give three methods (inverse transform method, acceptance-rejection method, and method based on Problem 8) to generate random number form the CDF

$$F(x) = x^n \quad \text{for} \quad 0 < x < 1.$$

    Implement these methods and discuss the efficiency of these three approaches.